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ONTARIO
Department of Education

Courses of Study

Grade XIII

MATHEMATICS

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Grade XIII

OUTLINE OF GENERAL COURSE

ALGEBRA

Arithmetic and geometric series with a brief account of the harmonic series; the infinite geometric series; the sum of squares of the natural numbers; related series.

Supplementary topics: the sum of the cubes of the natural numbers, the arithmetic-geometric series.

Ratio and proportion: if $\frac{a}{b}=\frac{c}{d}$ then each of these $=\frac{a+c}{b+d}$; solution of $ax+by+cz=0$, $a_1x+b_1y+c_1z=0$, $a_2x^2+b_2y^2+c_2z^2=d$.

Variation: definitions of "vary (directly) as," "vary inversely as," "vary jointly as"; treatment limited to the fundamental theorems and exercises thereon.

Functions: examples of functions defined by expressions, by statements, by graphs, by tables of values; formal definition of function.

Linear functions: their graphs; determination of a linear function from two known values; the linear function in the direct variation specified above.

Quadratic functions: their graphs; their factorization when they possess real factors; the values of the variable which produce positive, zero, negative, maximum or minimum values for the function; review of quadratic equations with real roots.

Number systems: brief discussion of the systems of whole numbers, signed integers, rational numbers, real numbers, complex numbers, and the laws of combination of the numbers of these systems.

Quadratic functions continued: the factorization of quadratic functions; quadratic equations; the discriminant; its use in determining the maximum or minimum value of a quadratic function; the sum and product and other simple symmetric functions of the roots.

Rational functions: equations; definitions of integral and of fractional rational functions of one variable; statements of theorems on the existence of roots of algebraic equations; remainder theorem and factor theorem; determination of a quadratic function from three known values; simple fractional rational functions such as $\frac{1}{x}$, $\frac{1}{x^2+1}$, $\frac{x}{x^2+1}$, their graphs and their properties; decomposition of simple functions, e.g. $\frac{3x-5}{(x-1)\ (x-2)}$ into $\frac{2}{x-1}+\frac{1}{x-2}$; solution of equations which depend upon quadratics and of equations for which a root may be discovered by inspection.

Supplementary topics: graphs of simple fractional rational functions, e.g. $\frac{3x-5}{(x-1)\ (x-2)}$; factoring by symmetry.

Definition of integral and of fractional rational functions of more than one variable; systems of linear equations; systems of two equations of which one is linear and one quadratic; simple systems of two quadratic equations.

Permutations and combinations: the usual topics, omitting consideration of the value of r for which ${}_nC_r$ is greatest.

The binomial theorem: its proof for positive integral indices, discussion for other real indices, with emphasis on sufficient conditions for the truth of the theorem. Customary topics which may be omitted are: the recognition of the binomial from which a given series is derived, the properties of the coefficients, the numerically greatest term, the number of homogeneous products.

Mathematics of investment: amount and present value of a sum of money and of an annuity; bonds, debentures, mortgages, sinking funds. In this topic the objective is an intelligent use of the geometric series rather than the development of formulas and symbols.

PLANE ANALYTIC GEOMETRY

The content of the course is as follows:

Section 1. **The Notion of Co-ordinates**

Axes of reference; Cartesian co-ordinates of a point.

Section 2. **First Applications of Co-ordinates**

Distance between points; internal and external division of a line segment in a given ratio; areas of triangles and other rectilinear figures in terms of the co-ordinates of the vertices.

Section 3. **Slope of a Line**

Definition; condition for parallelism and for perpendicularity of two lines.

Supplementary topic: angle between two lines.

Section 4. **Loci and Their Equations**

Locus defined as the path traced by a point which moves according to a given law; the equation of the locus as the algebraic statement of this law; drawing simple loci by (1) interpreting their equations, (2) plotting a number of points; the points of intersection of loci.

Section 5. **The Straight Line**

Various forms of equation of line (slope and one point, slope and y-intercept, two point, two intercept); general equation of line; slope of line whose equation is given; conditions for parallelism and perpendicu-

larity in terms of the coefficients; reduction of equation of line to desired form; families of lines.

Perpendicular distance from a point to a line; sign of $ax+by+c$ as (x, y) varies; bisectors of angle between two lines.

Supplementary topic: the normal form, and the direction cosine form, of the equation of a line; the equation of a pair of lines through the origin; the angle between these lines.

Section 6. **The Circle**

Equations of circle in standard forms; reduction of general equation of circle to standard form.

Definition of a tangent to a curve as the limit of a secant; equation of tangent to circle at a given point; tangent with given slope; equation of tangent from a given point in numerical cases only; length of tangent; radical axis.

Families of circles.

Section 7. **The Parabola**

Definition of parabola—the locus of a point which is equidistant from a fixed point and a fixed line.

Construction by (1) point-by-point method using ruler and compasses, (2) mechanical method using set-square and cord.

Development of equation $y^2=4px$; determination, from this equation, of intercepts, range of values of co-ordinates, symmetry; the number of points of intersection of a parabola with an arbitrary straight line; latus rectum; drawing of parabolas from the equations in numerical cases by plotting points.

Discussion of the equations $y^2=-4px$, $x^2=4py$, and $x^2=-4py$.

Equations of tangent and normal at a given point; tangent with a given slope; tangent from a given point in numerical cases; lengths of subnormal and subtangent; construction of tangent at a given point by ruler and compasses.

Proof of theorem: the normal to a parabola at any point bisects the angle between the focal radius to the point and the line through the point parallel to the axis of the parabola. Application of this theorem to the construction of parabolic mirrors in searchlights and reflecting telescopes.

Diameters of a parabola.

Examples of the occurrence of the parabola in physics.

Supplementary topic: the locus of the intersection of perpendicular tangents to a parabola is the directrix.

Section 8. **The Ellipse**

Definition—based on the constant sum of the focal distances.

Construction by (1) point-by-point method using ruler and compasses, (2) mechanical method using cord.

Development of the equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; determination, from this equation, of intercepts, range of values of co-ordinates, symmetry; the number of points of intersection of an ellipse with an arbitrary straight line; latus rectum; eccentricity.

Tangent and normal at a point on the ellipse; construction of tangent at a point on the ellipse by means of the circle on major (or minor) axis as diameter.

Diameters of an ellipse; conjugate diameters.

Determination of area of ellipse by comparison of ordinates of ellipse with those of circle on major (or minor) axis as diameter.

Supplementary topics: geometric constructions of ellipse from circles on major and minor axes; formulas $a+ex$ and $a-ex$ for focal distances; tangent and normal at a point bisect the angles between focal radii; definition of ellipse in terms of focus, directrix and eccentricity.

Section 9. **The Hyperbola**

Definition—based on constant difference of focal distances.

Construction by (1) point-by-point method using ruler and compasses, (2) mechanical method using cord.

Development of the equation $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$; determination, from this equation, of intercepts, range of values of co-ordinates, symmetry; the number of points of intersections of an hyperbola with an arbitrary straight line; latus rectum; eccentricity.

Discussion of the equations $y=\pm\frac{b}{a}\sqrt{x^2-a^2}$ for large values of x; asymptotes.

Discussion of the equation $\frac{x^2}{b^2}-\frac{y^2}{a^2}=-1$; conjugate hyperbola.

The equilateral hyperbola $x^2-y^2=a^2$; development of the equation $xy=\frac{a^2}{2}$ when the asymptotes are taken as axes of co-ordinates.

Examples of the occurrence of the equilateral hyperbola in physics.

Supplementary topics: application of hyperbolas in locating an invisible source of sound; formulas $ex-a$ and $ex+a$ for focal distances; definition of hyperbola in terms of focus, directrix and eccentricity; tangents and normals; diameters and conjugate diameters.

TRIGONOMETRY AND STATICS

Trigonometry

In this subject two objectives are to be kept in mind. One of these is the solution of triangles, a problem fundamental in surveying; the other is the development of analytic trigonometry, that is, the theory of the circular functions, as outlined in sections 3, 5, 8 and 10 of the following syllabus. It is to be noted that the first of these objectives can be accomplished independently of the second. All the formulas necessary for the solution of triangles, with or without the use of logarithms, can be obtained from geometry and the definitions of the trigonometric functions. Hence, if desired, the topics of sections 6 and 7 may be studied before those of section 5.

Section 1. Review of the trigonometry of previous grades with attention to the solution of right-angled triangles and applications; values of the trigonometric functions at 30°, 45° and 60°.

Section 2. Definitions of the trigonometric functions of positive and of negative angles of any size (these may be given in terms of the coördinates of a point on the unit circle); signs of the functions of angles belonging to each of the four quadrants; values of functions for such angles as 135°, 210°, found from diagrams. Proofs of the following formulas where A is an angle of any magnitude:

$\tan A = \frac{\sin A}{\cos A}$; $\cos^2 A + \sin^2 A = 1$; $\sec^2 A = 1 + \tan^2 A$; $\operatorname{cosec}^2 A = 1 + \cot^2 A$.

Section 3. Variation of the trigonometric functions. The values of the functions at 0°, 90°, 180°, 270°, where these values exist; the changes in sign and magnitude of sin x, cos x, tan x, as x increases from 0° to 360°; the periodicity and the graphs of sin x, cos x, tan x; the functions of —x, (90°±x), (180°±x), (270°±x), found from diagrams in terms of the functions of x.

Section 4. Review logarithms. Definition of $\log_a x$; range of values of x for which $\log_a x$ is defined; graph of $\log_a x$. Working rules of logarithms: $\log_a xy = \log_a x + \log_a y$; $\log_a \frac{x}{y} = \log_a x - \log_a y$; $\log_a x^p = p \log_a x$.
Computation with logarithms. Solution of exponential equations in one unknown.

Supplementary topic: the slide rule.

Section 5. Functions of the sum and of the difference of two angles (additional formulas). Sine and cosine of (A+B) and of (A—B), (proofs for the cases in which A, B, A+B, A—B are positive acute angles; discussion also for angles of any magnitude); derivation of tan (A+B) and of tan (A—B); values of the functions for such angles as 15°, 75°; practice in such expansions as those of sin (A+B+C), cos (A+B—C), tan (A+B+C); functions of 2A, 3A in terms of those of A; transformations of sums and differences into products and of products into sums and differences.

Supplementary topic: the functions of 18° and of its multiples.

Section 6. Some properties of triangles. The law of sines ($\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$); the law of cosines ($\cos A = \frac{b^2+c^2-a^2}{2bc}$); application of of these laws to the solution of triangles.

Section 7. The solution of triangles with the aid of logarithms. Formulas adapted to logarithms; the law of sines; the functions of the half-angles in terms of the sides (geometric or analytic proofs); the law of tangents, $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$ (geometric or analytic proof); application of these formulas to the solution of triangles; problems in heights and distances in one or more planes.

Supplementary topic: deduction of the other formulas for the triangle from the cosine formula.

Section 8. Radian measure. The relation $\frac{\text{circumference}}{\text{diameter}} = \text{constant}$; symbol for this constant and its approximate value. Definition of radian; radian measure of any angle $(\theta = \frac{a}{r})$; the formulas $a = r\theta$ and 180 degrees $= \pi$ radians.

Section 9. Further properties of triangles; regular polygons. Area of triangle; radius of circumscribed circle; radii of inscribed and escribed circles; perimeter and area of regular polygon, (i) inscribed in a circle, (ii) circumscribed about a circle; area of circle; area of sector of a circle.

Section 10. Inverse circular functions. The functions $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$; their graphs; formulas for all angles having a given sine, cosine, tangent; solution of easy trigonometric equations.

Statics

It is expected that wherever possible experimental demonstrations will accompany the teaching of the various topics.

1. Weight: the equal-arm balance; bodies of equal weight; the weight of a body in terms of the weight of a standard body.

2. Force as a concept associated with our sensations when lifting or pushing a body.
   Point of application, direction, and magnitude as elements of a force. The weights of bodies, at a place on the earth, as forces and as measures of forces.
   The spring balance: its calibration and its use in the measurement of force.
   The transmission of force by a string under tension and by a rod under compression.

3. Experimental study of the composition and resolution of forces pushing or pulling a particle. The parallelogram law. Definition of vector; expression of the parallelogram law in terms of vectors.
   Calculation of components and resultants.
   Resultant of two collinear and of two, three, or more coplanar but non-collinear forces acting on a particle.
   The triangle and polygon of forces.
   The conditions of equilibrium of a particle acted upon by two, three, or any number of forces.

4. The capacity of a solid body to transmit force with preservation of direction.
   Experimental study of levers, leading to the definition of the moment of a force about a point.
   The equality of the sum of the moments of two or more forces and the moment of their resultant.
   The composition of parallel forces. Couples.
   Centre of gravity: its determination by experiment and, for bodies of simple shape, by calculation.

5. Newton's Third Law—when one body exerts a force on a second the second exerts a force on the first equal in magnitude and opposite in direction.

6. The reduction of a set of coplanar forces acting on a lamina to a simple set of forces.
   The conditions for the equilibrium of a lamina.

7. Friction: illustrative examples; laws of friction; coefficient of friction.
   Direction of the reaction of a smooth surface.

8. Machines: mechanical advantage; lever, pulley, wheel and axle, train of toothed wheels, inclined plane, wedge, screw.

9. Description of Von Jolly's experiment. The adequacy or inadequacy of the weight of a standard body as the unit of force.